To mix or not to mix? Diffusion and segregation in heterogeneous groups

Segismundo S. Izquierdo. Universidad de Valladolid. Spain
Luis R. Izquierdo. Universidad de Burgos. Spain
Dunia López-Pintado. Universidad Pablo de Olavide. Spain

Consider a population of individuals who can be either *infected* or *not infected*. The *infected* state may represent catching a disease, but it may also be interpreted in the broader context of contagion processes as e.g. the adoption of the latest technology, or the state of active participation in the classroom. Individuals abandon the infected state (i.e., become healthy) with a certain probability, and may become infected again (or adopt the latest technology, or gain motivation in the classroom) if they meet an infected individual. Some individuals are less prone to be infected than other individuals –either because they have a lower propensity to catch the disease when meeting an infected individual or because they enjoy a higher propensity to heal when they are infected. These different individual propensities define two groups: the *sensitive* group and the *resistant* group (see Fig. 1). The *sensitive* group (red) has a greater natural propensity to be infected than the *resistant* group (blue), which naturally presents a lower (dynamically stable) infection level.

In this paper, we analyse the consequences of increasing the level of mixing between the groups (i.e., the fraction of links between individuals belonging to different groups), while keeping constant the average level of individual interaction. One might intuitively predict that the infection levels in both groups should approximate, with an increase of infections in the resistant group and a decrease of infections in the sensitive group (Fig. 1, case A). However, we find situations for which an increase in between-group interaction leads to non-monotonic effects in one of the groups and, possibly, to a reduction of infections in *both* groups (Fig. 1, case B). This implies that mixing can help both groups achieve lower rates of infection, and therefore, that some levels of mixing are Pareto inefficient.

The underlying reason for this paradoxical effect is the feedback loop created between groups: from an initial stable situation corresponding to a given mixing level (e.g., 60% mixing in the scenario corresponding to case B in Fig. 1), increasing the interaction level between groups can be initially costly for the resistant group, which will initially meet more infected individuals. However, it can turn out to be beneficial for that same group once the returns from the positive effect induced on the other group are collected, and a new equilibrium is obtained in which both groups are better off.

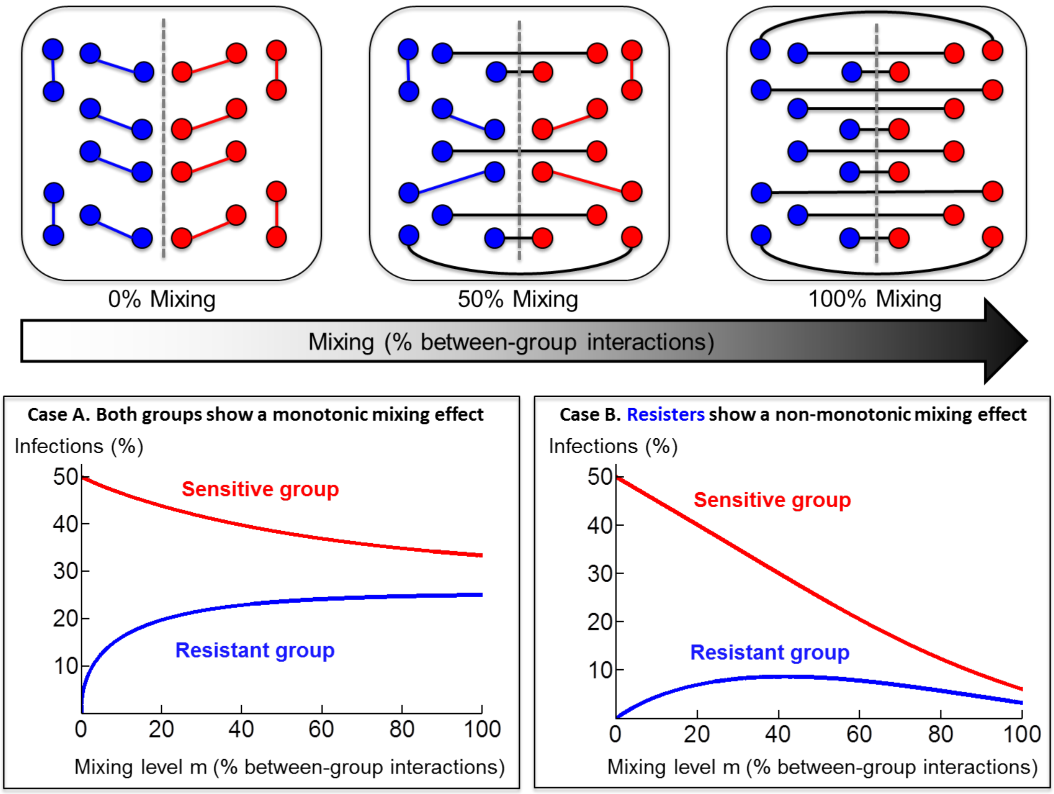

Fig. 1. Interaction structure and adoption levels as a function of mixing. Top, from left to right: segregated population case (*mixing* = 0), unbiasedly mixed population case (*mixing* = 0.5) and bipartite population case (*mixing* = 1). Bottom: Adoption levels in equilibrium for the resistant group (blue) and the sensitive group (red) as a function of the mixing level for two different cases. In case A, the adoption levels are monotonic functions of *mixing*, and all mixing levels are Pareto efficient. In case B, the adoption level in the resistant group is a non-monotonic function of the mixing level, and Pareto inefficient mixing levels exist.