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Dimensional fragility of the Kardar–Parisi–Zhang universality class

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Abstract. We assess the dependence on substrate dimensionality of the asymptotic scaling behavior of a whole family of equations that feature the basic symmetries of the Kardar–Parisi–Zhang (KPZ) equation. Even for cases in which, as expected from universality arguments, these models display KPZ values for the critical exponents and limit distributions, their behavior deviates from KPZ scaling for increasing system dimensions. Such a fragility of KPZ universality contradicts naive expectations, and questions straightforward application of universality principles for the continuum description of experimental systems.

Keywords: kinetic growth processes (theory), kinetic roughening (theory), self-affine roughness (theory)

One of the most powerful concepts in contemporary statistical mechanics is the idea of universality, by which microscopically dissimilar systems show the same large scale behavior, provided they are controlled by interactions that share dimensionality, symmetries, and conservation laws. Being rooted in the behavior of equilibrium critical systems [1], universality has more recently made it possible to describe scaling behavior far from equilibrium [2]–[4], such as for, for example, the stock market [5], crackling-noise [6], or random networks [7]. In complex systems like these, universality provides an enormously simplifying framework, as significant descriptions can be put forward on the basis of the general principles just mentioned.

Celebrated non-equilibrium systems include those with generic scale invariance, displaying criticality throughout parameter space [8]. Examples are self-organized-critical [9] and driven-diffusive systems [10], or surface kinetic roughening [11]. Indeed, the paradigmatic Kardar–Parisi–Zhang (KPZ) equation for a rough interface [12]

$$\partial_t h = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta(\mathbf{x}, t), \tag{1}$$

has very recently proved itself as a remarkable instance of universality. Here, $h(\mathbf{x}, t)$ is a height field above substrate position $\mathbf{x} \in \mathbb{R}^d$ at time t, and η is Gaussian white noise with zero mean and variance $2D$. The exact asymptotic height distribution function has been very recently obtained for $d = 1$ [13]–[15]: it is given by the largest-eigenvalue distribution of large random matrices in the Gaussian unitary (GUE) (orthogonal, GOE) ensemble, the Tracy–Widom (TW) distribution, for globally curved (flat) interfaces, as proposed in [16], see reviews in [17, 18]. Besides elucidating fascinating connections with probabilistic and exactly solvable systems, these results show that, not only are the critical exponent values common to members of this universality class, but also the distribution functions and limiting processes are shared by discrete models and continuum equations [19], and by experimental systems, from turbulent liquid crystals [20] to drying colloidal suspensions [21, 22].

In view of the success for $d = 1$ (1D) substrates, a natural important step is to assess the behavior of the KPZ universality class when changing space dimension, analogous to, for example, the experimental change from 2D to 1D behavior for ferromagnetic nanowires, that nonetheless occurs within the creeping-domain-wall class [23]. For discrete models and the continuous equation itself, universal distributions have been also very recently found [24, 25] to control height fluctuations in the $d = 2$ KPZ class, providing analogs of the TW distributions. Again this underscores universality—beyond the critical exponents already known to be shared by the KPZ equation, many discrete models [26], and some experiments [27]—although much less than expected [28]. This fact calls for further experimental verification, akin to that recently provided [20]–[22] for the one-dimensional case.

In this work we report a fragility of the KPZ universality class with respect to space dimension. Namely, we consider a family of continuum equations with the symmetries and conservation laws of the KPZ equation. We provide conditions under which, although the system is accurately described by KPZ universality in $d = 1$, the scaling exponents *depart* from the latter in $d = 2$. Analogous behavior had been found earlier for discrete models of conservative surface growth [29]—namely, a change of the universality class of a given system with d. Here we demonstrate it for non-conserved dynamics, and at the level of continuum equations. Note that this is not a change in the universality class as a response to changes in appropriate system parameters for a fixed dimension, as seen e.g. for Barkhausen criticality [30]. The lack of universality that we find signals a serious difficulty in the identification of the appropriate universality class for experimental systems, by preventing cursory use of universality arguments to propose theoretical descriptions,

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stressing the need for physically motivated models [28]. This should be borne in mind in view of the timely interest in the experimental validation of 2D KPZ universality.

We consider the following equation [31, 32]:

$$\partial_t h_{\mathbf{k}}(t) = (\nu k^\mu - \mathcal{K}k^2)h_{\mathbf{k}}(t) + \frac{\lambda}{2}\mathcal{F}[(\nabla h)^2]_{\mathbf{k}} + \eta_{\mathbf{k}}(t), \tag{2}$$

where $\nu, \mathcal{K} > 0$, $k = |\mathbf{k}|$, and $\mathcal{F}$ is space Fourier transform, with $h_{\mathbf{k}}(t)$ and $\eta_{\mathbf{k}}(t)$ being the $\mathbf{k}$th modes of the height and noise fields, respectively. Equation (2) perturbs the KPZ equation (1) through the linear term with coefficient ν, where $0 < \mu < 2$. This family of equations includes celebrated systems, such as the Kuramoto–Sivashinsky (KS) (take $\mu \to 2$ and replace k^2 with k^4) and the Michelson–Sivashinsky (MS) ($\mu = 1$) equations [31], that combine pattern formation at short scales with asymptotic kinetic roughening [33]. Indeed, the positive sign of the $\nu k^\mu h_{\mathbf{k}}(t)$ term in equation (2) induces rapid growth of the height modes, reflecting a morphological instability, while the negative sign of the remaining linear contribution, $-\mathcal{K}k^2 h_{\mathbf{k}}(t)$, induces attenuation of surface features, counteracting the instability. The competition between both contributions to the so-called linear dispersion relation [34] $\sigma_{\mathbf{k}} = \nu k^\mu - \mathcal{K}k^2$ leads to selection of a typical length scale, that is $\ell = 2\pi/k_*$, where $k_* = |\mathbf{k}_*|$ corresponds to that wavevector $\mathbf{k}_*$ for which $\sigma_{\mathbf{k}}$ takes its *positive* maximum value. Physically, the origin of the unstable νk^μ is with competitive effects whereby parts of a growing interface which are more exposed to transport of material grow faster than the parts which are less exposed, height variations among them being amplified. A paradigmatic example is provided by diffusion-limited growth, see [28] and references therein: here, the unstable contribution implements the well-known tip effect by which surface maxima of a growing cluster capture more efficiently particles from the diffusive currents, growing faster than surface minima. The stabilizing term in $\sigma_{\mathbf{k}}$ is on the other hand associated with surface tension effects that smooth out surface features at small scales.

Non-linear equations with the precise shape of equation (2) have been explicitly derived from physical models in a number of contexts, usually within weakly nonlinear approximations of the corresponding moving boundary descriptions. Take for instance the KS equation as a model of an isolated step on a vicinal epitaxial surface [33], for which $\mu = 2$, or the description by the MS equation, for which $\mu = 1$, of the so-called Darrieus–Landau instability in the propagation of a premixed laminar flame [35]. Actually, instances of equation (2) for appropriate μ values do provide *quantitatively* accurate descriptions of specific experiments for a number of different interface processes in which dynamics are transport-limited, like plasma etching [36] ($\mu = 1$), electrochemical deposition [37] ($\mu = 0.75$), and chemical vapor deposition [38] ($\mu = 1$).

Equation (2) complies with the standard symmetries of the KPZ class. Namely, dynamics are non-conserved, isotropic and reflection invariant in $\mathbf{x}$, invariant under Galilean transformations and under arbitrary shifts $h \to h + \text{const.}$, and the up–down symmetry $h \leftrightarrow -h$ is broken [32]. Two additional features are to be noted—namely, the non-analytic dependence [39] on $\mathbf{k}$, and the morphological instability [28]. The former induces *non-locality* of the equation when written in real space[4,5]. The latter leads,

[4] The real space representation of $k^\mu h_{\mathbf{k}}$ for $0 < \mu \le 2$ is proportional [32] to the Cauchy principal value of $\int_{\mathbb{R}^d} [h(\mathbf{r}) - h(\mathbf{r}')]/|\mathbf{r} - \mathbf{r}'|^{d+\mu}\,\mathrm{d}\mathbf{r}'$.

[5] Equation (2) is a non-equilibrium system with weakly long-range interactions, see [40].

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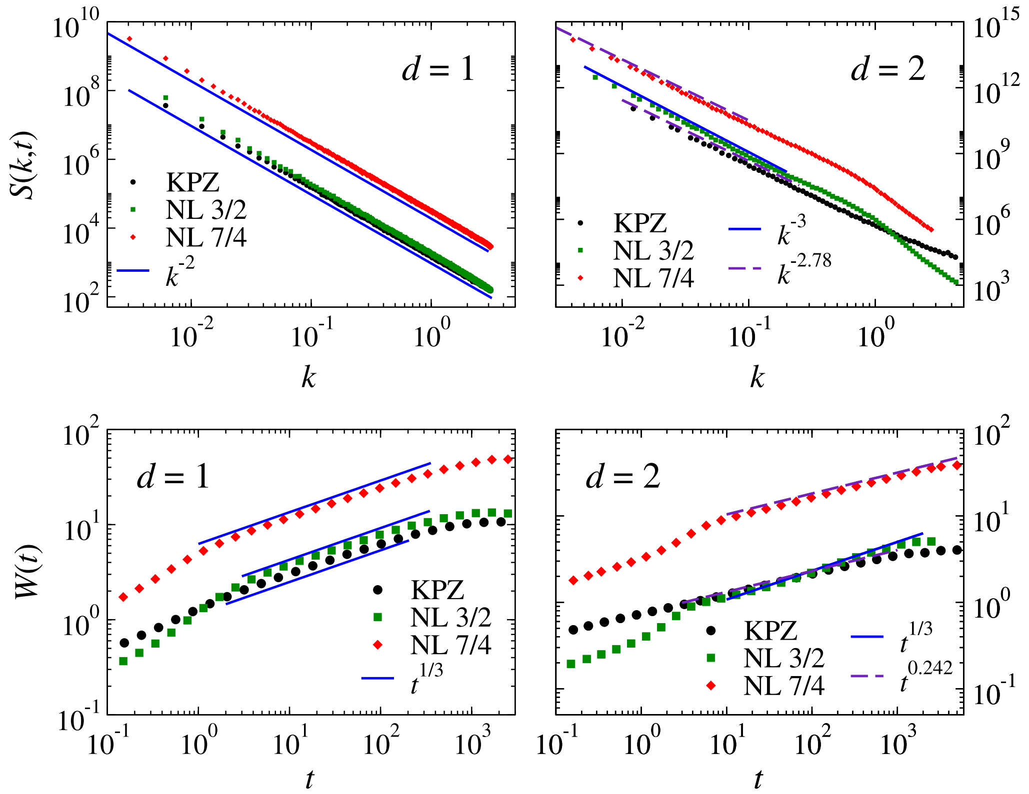

Figure 1. Numerical simulations of equation (1) (●) and equation (2) for $\mu = 3/2$ (■) and $\mu = 7/4$ (♦). Surface structure factor (upper row) and roughness (lower row) for $d = 1$ (left column) and $d = 2$ (right column). Solid and dashed lines represent power-law behaviors as indicated, which are discussed in the main text. All the observables have been averaged over 10^3 (10^2) different noise realizations for $d = 1$ ($d = 2$), starting from a flat initial condition. Error bars are smaller than symbol sizes. All units are arbitrary.

as mentioned above, to selection of a preferred length scale ℓ at short times during which dynamics are governed by the linear dispersion relation $\sigma_{\mathbf{k}}$, and thus to a loss of scale invariance. Nevertheless, kinetic roughening is restored back along the dynamics at large enough time and length scales, as in the KS system [11]. Indeed, as borne out by numerical [31] and dynamic renormalization group [32] results, the asymptotic behavior of equation (2) fulfils the Family–Vicsek (FV) scaling ansatz [11]. Hence, the surface structure factor, $S(k,t) = \langle |h_{\mathbf{k}}(t)|^2 \rangle$, scales at long times as $S(k, t \to \infty) \sim 1/k^{2\alpha+d}$, with a well-defined value of the roughness exponent α. The crossover wavevector value separating white noise from correlated behavior also scales, $k_c \sim t^{-1/z}$, leading to power-law behavior of the global roughness $W(t)$ (root mean square fluctuation of the surface height) with time as $W \sim t^{\beta}$, with $\beta = \alpha/z$. The values of these critical exponents depend on μ, and correspond to the KPZ universality class in d dimensions, provided $z_{\mathrm{KPZ}}(d) \le \mu < 2$, where $z_{\mathrm{KPZ}}(d)$ is the corresponding KPZ value of the dynamic exponent. For $\mu < z_{\mathrm{KPZ}}(d)$ and any d, the asymptotic exponents are non-KPZ, namely, $z = \mu$ and $\alpha = 2 - z$ [31]. Note that, for the morphologically unstable condition $\nu > 0$ that we consider, the nonlinearity is dynamically relevant for any value of μ (even if it may not control scaling behavior),

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Table 1. Parameters used for the numerical integrations reported in this work. NL stands for the non-local models, i.e. equation (2) with μ equal to 3/2 or 7/4. L is the size of the 1D domain, or the edge of the 2D square, used for simulations in figure 1. All units are arbitrary.

	Equation	ν	$\mathcal{K}$	λ	D	L	Δx	Δt
$d=1$	KPZ	1		5.0	1.46	1024	1.0	0.002
	NL 3/2	1	1.0	5.0	0.50	1024	1.0	0.001
	NL 7/4	1	1.7	2.5	12.5	2048	2.0	0.002
$d=2$	KPZ	1		2.5	1.46	512	1.0	0.01
	NL 3/2	1	0.5	10	0.50	1024	1.0	0.01
	NL 7/4	1	1.0	1.0	50.0	1536	1.5	0.02

Table 2. Values of the constants used for the determination of the probability distribution function $P(\chi)$. Here, t^* is the time used in the computation of the quantities reported in figures 3 and 4. The system size for these simulations is $L = 65\,536$. In the last column we report the absolute error between calculated coefficient $c_v = \beta\Gamma^\beta\langle\chi\rangle$ and its estimated value $c_v^{\rm est}$ from our numerical data for the three equations in the case of one-dimensional substrates ($d = 1$).

	Equation	v_∞	Γ	t^*	$c_v^{\rm est}$	c_v	Absolute error (%)
$d=1$	KPZ	3.6990	5.620	1000	0.45	0.450 45	0.01
	NL 3/2	4.2675	11.95	500	0.52	0.579 23	10
	NL 7/4	18.4625	310.0	500	1.56	1.714 69	9

while for the morphologically stable situation ($\nu < 0$) scaling in equation (2) is controlled by the linear terms for small μ values [31, 32, 41].

In the left column of figure 1 we show 1D numerical simulations of equation (2) for $\mu = 3/2, 7/4$ (denoted as NL, for non-local) and, as a reference, for the KPZ equation itself, using a pseudo-spectral scheme as in [31, 37] and parameters reported in table 1. Both values of $\mu \geq z_{\rm KPZ}(1) = 3/2$, thus for $d = 1$ the behavior is well described by KPZ exponents $\beta_{\rm KPZ}(1) = 1/3$ and $\alpha_{\rm KPZ}(1) = 1/2$. 1D simulations with non-KPZ exponents for $\mu < z_{\rm KPZ}(d)$ can be found in [31].

KPZ universality here *goes beyond* exponent values. Thus, using the Ansatz [16]

$$h(x,t) \simeq v_\infty t + \mathrm{sgn}(\lambda)(\Gamma t)^\beta \chi, \tag{3}$$

we can measure the fluctuations of the interface around its mean value $v_\infty t$, i.e. $\chi = \mathrm{sgn}(\lambda)(h - v_\infty t)/(\Gamma t)^\beta$. Through Γ we normalize the variance of this stochastic process to the variance of the TW–GOE distribution and we are able to compare them. For the estimation of v_∞ and Γ we followed the procedure described in [19]. Values for these constants are reported in table 2. Specifically, Γ and v_∞ are calculated by averaging quantities measured on interface profiles $h(x,t)$. Thus, after time differentiation of equation (3), we obtain an Ansatz for the instantaneous velocity of each point of the interface,

$$v(x,t) \simeq v_\infty + \beta\,\mathrm{sgn}(\lambda)\Gamma^\beta t^{\beta-1}\,\chi, \tag{4}$$

so that the average of this observable reads

$$\langle \bar{v} \rangle = \frac{\mathrm{d}}{\mathrm{d}t}\langle \bar{h} \rangle = v_\infty + \beta\, \mathrm{sgn}(\lambda)\Gamma^\beta t^{\beta-1}\, \langle \chi \rangle, \tag{5}$$

provided $\langle \chi \rangle \neq 0$, where the overline stands for spatial averages on the same interface, while brackets refer to average over different runs. Plots $\langle \bar{h} \rangle$ versus t or $\langle \bar{v} \rangle$ versus $t^{\beta-1}$ are used to measure v_∞, depending on each equation we consider. Comparison of $P(\chi)$ with the TW–GOE distribution is possible only after we estimate v_∞ very accurately (up to the fourth decimal place); small errors result in a misalignment of the maxima of the two distributions. The second step is to normalize the variance of χ to the variance of the TW–GOE distribution by using the parameter Γ. From the variance of h (that is, the second cumulant $\langle h^2 \rangle_{\mathrm{c}}$) we cancel the $v_\infty t$ contribution so that $\langle h^2 \rangle_{\mathrm{c}} \simeq (\Gamma t)^{2\beta} \langle \chi^2 \rangle_{\mathrm{c}}$, and, by normalizing the variance of χ to $\langle \chi^2_{\mathrm{GOE}} \rangle_{\mathrm{c}} = 0.638$, we get

$$\Gamma = t^{-1}\left(\frac{\langle h^2 \rangle_{\mathrm{c}}}{\langle \chi^2_{\mathrm{GOE}} \rangle_{\mathrm{c}}}\right)^{1/2\beta}. \tag{6}$$

We show in figure 2 the implementation of this procedure to estimate v_∞ and Γ for the one-dimensional KPZ and non-local ($\mu = 3/2, 7/4$) equations, leading to the values quoted in table 2. Moreover, we perform a consistency check of equation (5) through the independent estimation of v_∞ and Γ. Namely, the Γ-dependent quantity $c_v(\Gamma) = \beta\Gamma^\beta \langle \chi \rangle$ calculated from Γ must match the coefficient of the time behavior of $\langle v \rangle$ obtained from numerical data. This calculation is straightforward for $d = 1$ equations where the limit distribution χ is known and $\langle \chi \rangle = -0.760\,07$, see table 2. For the KPZ equation the agreement is remarkable, but for non-local equations errors are not so small. For these equations, the transient regime between the initial exponential growth due to the pattern formation process and the asymptotic scale invariant state hinders a clearer match; probably, longer times are required in order to get a better agreement between $c_v(\Gamma)$ and the coefficient from $\langle v \rangle$ data. Still, the red dashed lines plotted in figure 2 show that even a fit with the calculated coefficient c_v (with $\sim 10\%$ absolute error) is compatible with our numerical data.

Once v_∞ and Γ have been determined, we can assess the probability distribution for the rescaled height fluctuations χ in each one of the equations we are studying. Numerical results are provided in figure 3, where they are compared with the analytical GOE–TW distribution, shown as a solid line. The latter has been calculated from the solution of the Painlevé II differential equation [42], and normalized according to [16]. Clearly, the $P(\chi)$ distribution is time-independent and indeed agrees with the analytical result for the three equations, providing in particular an assessment of the TW distribution also for non-local equations. Further universal behavior is seen to occur, as in other instances of the 1D KPZ class [19], for the two-point correlation $C(x,t) = \langle h(x_0 + x, t)h(x_0, t)\rangle - \langle h \rangle^2$, that scales as $C(x,t) \sim (2\Gamma t)^{2/3} g_1(u)$, where $u = (Ax/2)/(2\Gamma t)^{2/3}$ with $g_1(u)$ the covariance of the Airy$_1$ process [19, 43], and $A = (2\Gamma/\lambda)^{1/2}$ for continuum equations (for discrete models, A is estimated from the local roughness [19]). As suggested by figure 3, even for the 1D non-local models the rescaled two-point correlation function collapses perfectly onto $g_1(u)$, see figure 4.

Hence, in $d = 1$ all the strong universal properties of the KPZ class occur in the NL $\mu = 3/2$ and $7/4$ systems. However, when we increase the system dimension to

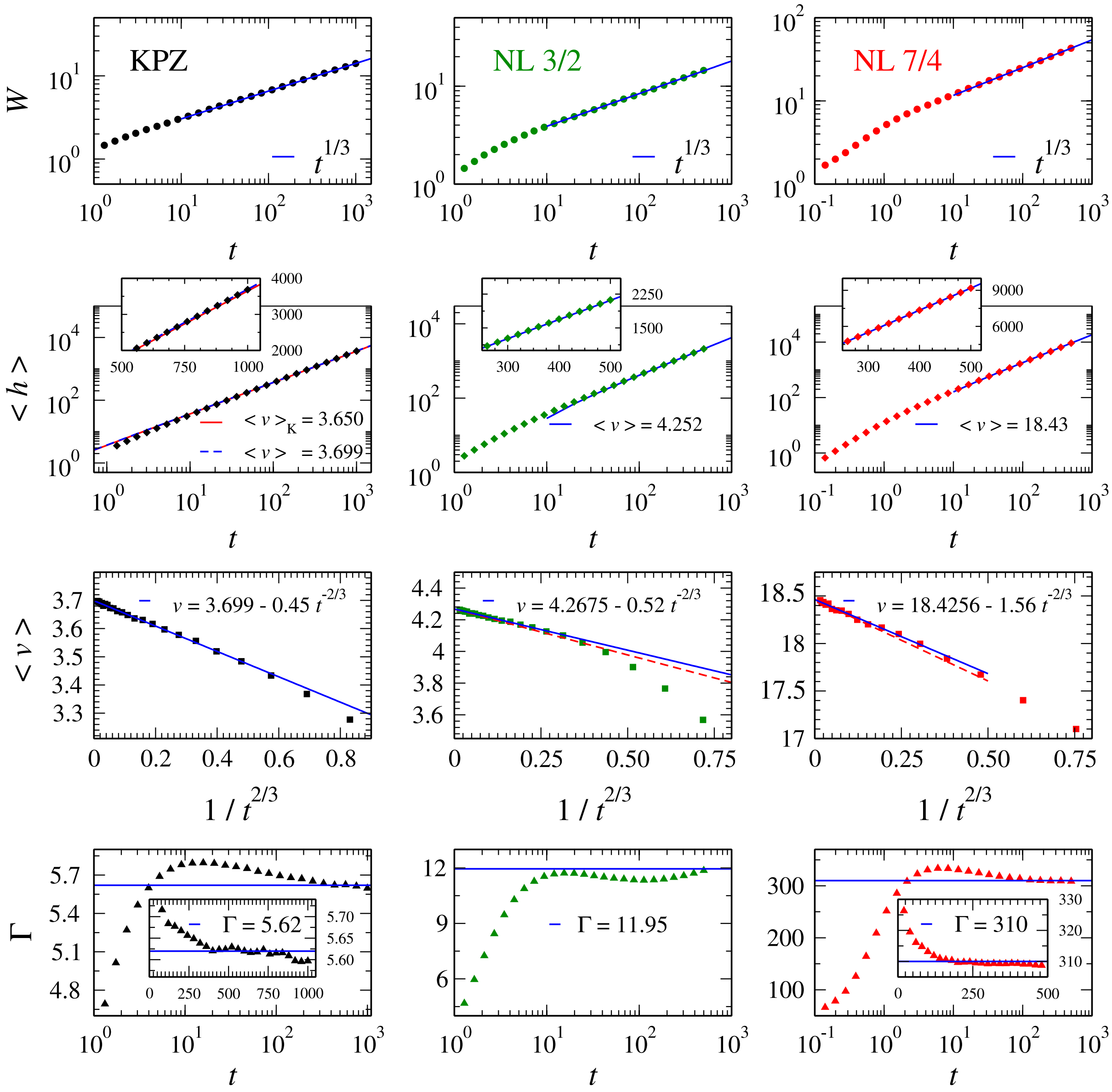

Figure 2. Estimation of v_∞ and Γ for the one-dimensional KPZ and non-local ($\mu = 3/2, 7/4$) equations. For the former, the value of v_∞ is equally well measured from $\langle \bar{h} \rangle$ and $\langle \bar{v} \rangle$ data, the red solid line in the $\langle h \rangle$ plot for the KPZ equation being the value of v_∞ in the limit of an infinite system size $\langle \bar{v} \rangle_{\mathrm{K}} = D\lambda/2\Delta x$ (with $\Delta x = 1$), calculated in [11] (note that our noise variance is $2D$ and not D). For the NL model, the value of v_∞ differs if considered from $\langle \bar{h} \rangle$ or from $\langle \bar{v} \rangle$ data. Here we take the latter choice because the $\langle \bar{v} \rangle$ data are smooth and do not display large fluctuations. The red dashed lines in the corresponding $\langle \bar{v} \rangle$ versus $1/t^{2/3}$ plots implement the consistency fit as calculated from the parameters reported in table 1.

$d = 2$, a remarkable departure from KPZ scaling occurs that depends on the relative values of μ and $z_{\mathrm{KPZ}}(2) \approx 1.61$. Thus, while equation (2) is still well described by KPZ exponent values for 'large' $\mu = 7/4 > 1.61$ as deduced from figure 1, namely $\alpha \simeq 0.39$

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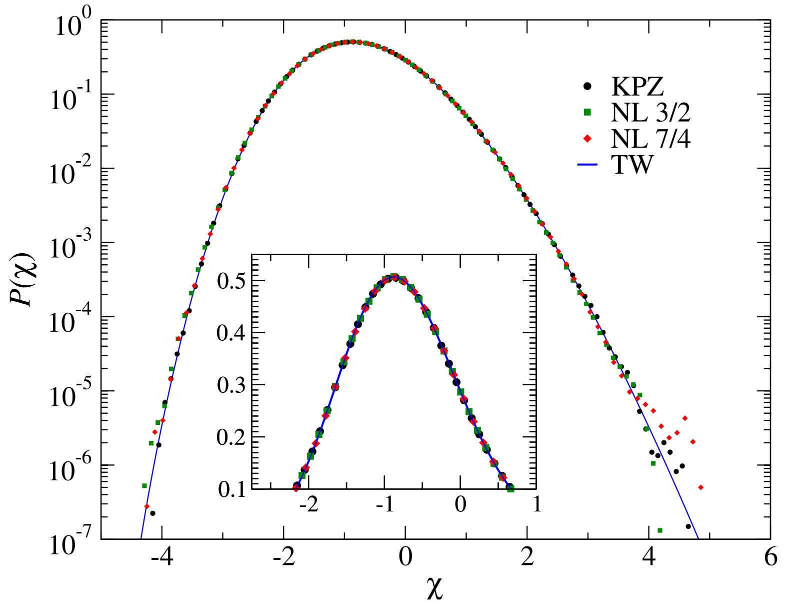

Figure 3. 1D height distributions for equation (1) (●) and equation (2) for $\mu = 3/2$ (■) and $\mu = 7/4$ (♦). The variable χ is defined in the text. The solid blue line is the TW (GOE) distribution expected for $d = 1$ [42]. $P(\chi)$ is estimated from 2048 independent runs starting from a flat initial condition. Inset: zoom of main panel, in linear representation. All units are arbitrary.

(compared with $\alpha_{\rm KPZ}(2) \approx 0.39$ [26]) and $z \simeq 1.61$, the 'small' $\mu = 3/2 < 1.61$ system has the same exponent values as for $d = 1$! Recall that, for $z_{\rm KPZ}(1) \leq \mu < z_{\rm KPZ}(2)$, the 2D exponents are non-KPZ [31], $z = \mu$, $\alpha = 2 - z$, moreover they are d-independent. Curiously enough, thus the $\mu = 3/2$ equation provides a peculiar example of a 2D system with 1D-KPZ exponents! Without the need of further characterization of height distributions[6] or correlation functions for this equation, this implies a change of its universality class as dimensionality increases from $d = 1$ to 2, while this is not the case for, for example, the $\mu = 7/4$ equation, which is still KPZ-like in 2D.

This fact has important consequences for the continuum modeling of systems, in particular of an experimental type, that are presumably in the KPZ universality class. Take the 1D case as an example. Equation (2) having the same symmetries as the KPZ equation, one might postulate the latter as a model description for a given experiment. But suppose the actual physical interactions lead to the occurrence of morphological instabilities (as happens only too often in surface growth experiments [28]), in such a way that a better

[6] In principle, the pseudo-spectral scheme we employed is known to outperform [44] the accuracy of the real space Euler algorithm used in [24] in reproducing a number of properties of the KPZ equation, like bare versus effective parameters, etc. Thus, it is interesting to compare the universal properties of $P(\chi)$ obtained through this scheme against previous estimates [24, 25]. As demonstrated in [25], the scaled cumulants g_n ($n \geq 1$) allow one to obtain universal quantities of $P(\chi)$, such as the ratios $R = g_2/g_1^2$, $S = g_3/g_2^{3/2}$ (skewness), and $K = g_4/g_2^2$ (excess kurtosis). For a 2D KPZ equation with parameters as in table 1, but $L = 1024$ and time $t^* = 250$, we obtain $g_1 = -1.149$, $g_2 = 0.4987$, $g_3 = 0.1578$, and $g_4 = 0.0941$, and universal ratios $R = 0.378$, $S = 0.448$, and $K = 0.378$, slightly larger (differences in the second or third significant digit) than those reported so far in the literature. This discrepancy can possibly be attributed to an incomplete convergence of the scaled cumulants for the chosen integration time t^*. A more accurate estimation of the height distribution for the 2D KPZ equation by means of the pseudo-spectral scheme is beyond the scope of this work, and will addressed in the future.

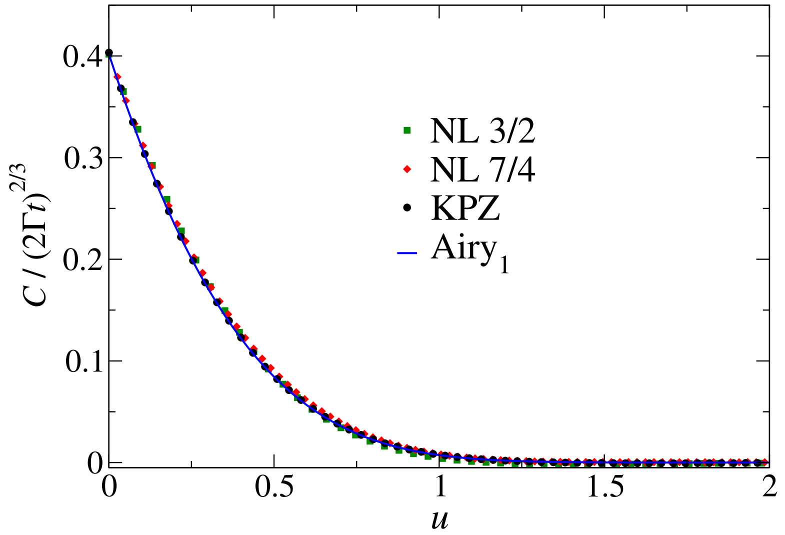

Figure 4. 1D height–height correlation function for equation (1) (●) and equation (2) for $\mu = 3/2$ (■) and $\mu = 7/4$ (♦). Here, $u = x\sqrt{\Gamma/2\lambda}/(2\Gamma t^*)^{2/3}$ while $C(x, t = t^*)$ is measured from the same surfaces employed to estimate $P(\chi)$ in figure 3. The solid blue line provides the covariance of the Airy$_1$ process [43]. All units are arbitrary.

description is provided by equation (2) for $\mu = 3/2$. This will not change the 1D scaling behavior with respect to KPZ universality, even at the level of height distributions or correlation functions. However, if one were able to perform an experiment for the 2D generalization of the system, a departure from KPZ behavior would be obtained, with the conclusion that the universality class of the physical system would *not* be KPZ. One might argue that increasing d for a fixed μ makes interactions more non-local in real space [40], and that the present fragility of KPZ scaling is only superficial [4]. However, this does not circumvent the need, for a given physical system, to assess in detail the occurrence of, for example, morphological instabilities and/or the range of interactions, in order to argue for the correct universality class on a safe basis. In any case, this requires going beyond symmetry principles to provide the sought-for continuum description. Note that, starting out with a higher value of μ that leads to KPZ scaling both in $d = 1$ and 2, such as $\mu = 7/4$, only pushes departure from KPZ scaling up to a higher dimension $d_{7/4}$, such that $z_{\rm KPZ}(d_{7/4}) > 7/4$, which will occur below the upper (if finite) critical dimension $d_{\rm c}$ for the KPZ universality class, at which $z_{\rm KPZ}(d_{\rm c}) = 2$.

Summarizing, we have found a fragility of the KPZ universality class with respect to space dimension, when perturbed by morphological instabilities combined with non-local interactions, within the *experimentally substantiated* family of equations, equation (2). Note that an important perturbation of the KPZ equation by instabilities that respects its space symmetries is also known to occur in the celebrated (noisy) KS system, which is a local equation known to lead to KPZ scaling, both in $d = 1$ [11] and $d = 2$ [45]. We recall that earlier results have also suggested non-universal behavior for the KPZ class in $d > 1$. For example, for increasing d, details of the noise distribution have been reported to become relevant [46]. Important issues remain open with respect to the behavior of the KPZ class in higher dimensions, like the existence and value of an upper critical dimension (see e.g. [47] and references therein), or even making mathematical sense of solutions to equation (1) [18].

Although non-equilibrium universality classes are frequently expected to be more fragile than equilibrium ones [2], a highly non-trivial question is to identify, if existent, the type of perturbation that is taking place here and assess its actual importance [4]. In our case, even if the present fragility might be relativized in view of the non-local nature of the perturbation, its occurrence is not easily circumvented by the symmetry arguments that are usually in use for the theoretical description of kinetic roughening phenomena. This seems an important *caveat*, especially in view of the current quest for 2D KPZ scaling behavior in *experimental* systems. As implied by our results, in order to assign a universality class to a given system, one would need to explore its behavior under a change in d. However, for many experimental systems modifying the space dimension may be hard to achieve without significantly altering the basic interactions that take place. For instance, basic properties of fluid flow can drastically change from a quasi-2D Hele-Shaw cell to a 3D system, while keeping all additional conditions unchanged [48]. This stresses the need for detailed modeling of the specific peculiarities of the system under study, undermining the promise of universality as the main toolbox for kinetically rough systems. An analogous situation occurs in the context of pattern formation, where Goldstone modes associated with the shift symmetry $h \to h + \text{const.}$ prevent the existence of a universal amplitude equation [34]. In such contexts, modeling has to be done on a system-specific basis. We note that in these cases symmetry arguments can be enhanced by multiple scale approaches in order to put forward general continuum models that successfully describe experimental systems [49]. One can ponder [39] whether analogous generalized approaches would be successful in the presence of non-localities and noise.

Acknowledgments

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